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FREE VIBRATION OF LINE SUPPORTED RECTANGULAR PLATES USING A SET OF STATIC BEAM FUNCTIONS

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The free vibration of thin orthotropic rectangular plates, which may be continuous over a number of intermediate line supports in one or two directions, is analyzed by the Rayleigh–Ritz method. A new set of admissible functions which are the static solutions of a point supported beam under a series of sine loads is developed. The eigenfrequency equation for the plate is derived by minimizing the potential energy. A very simple and general computer programme has been compiled. The basic concept to form the set of static beam functions is very clear and requires no complicated mathematical knowledge. Some numerical results presented are compared with those obtained by other numerical methods in the literature. It is shown that this set of static beam functions has some advantages in terms of computational cost, application versatility and numerical accuracy, especially for the plate problem with a large number of intermediate line supports and/or when higher vibrating modes need to be calculated.

1. INTRODUCTION

Vibration characteristics of rectangular plates with intermediate line supports in one or two directions (such plates are also called continuous plates) are of practical interest, since many applications of such structures are found in civil, naval, aerospace engineering and the like.

Much work has been devoted to the vibration of rectangular plates with intermediate line supports in one direction. Early research mainly focused on rectangular plates simply supported at two opposite edges and continuous over line supports perpendicular to those edges. Veletsos and Newmark [1] used the Holzer's method, Ungar [2] used a semi-graphical approach, Bolotin [3] and Moskalenko and Chien [4] used the dynamic edge-effect method, Lin *et al.* [5]

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used the transfer matrix method, Elishakoff and Sternberg [6] used the modified Bolotin's method, and Azimi *et al.* [7] used the receptance method for such plates. Furthermore, Cheung and Cheung [8] used the single-span vibrating beam functions in the finite strip method and Mizusawa and Kajita [9] used the B-spline functions in the Rayleigh–Ritz method to analyze the free vibration of one-direction continuous plates with arbitrary boundary conditions.

In the last two decades, the vibration of line supported rectangular plates which are continuous in two directions has received a lot of attention. Takahashi and Chishaki [10] used a sine series analytical solution for the vibration of rectangular plates with all edges simply supported and over a number of line supports in two directions. Wu and Cheung [11] used the multi-span vibrating beam functions to analyze the free vibration of continuous rectangular plates in one or two directions by the finite strip method. Kim and Dickinson [12] used a set of one-dimensional orthogonal polynomial functions to analyze the free vibration of line supported plates and plate systems by the Rayleigh–Ritz method. Furthermore, Liew and Lam [13] used a set of two-dimensional orthogonal polynomial functions to determine the eigenfrequencies of multi-span plates. Zhou [14] proposed a set of trial functions which are the single-span vibrating beam functions plus augmented polynomials to study the vibration of plates continuous in one or two directions, and Kong and Cheung [15] combined this set of trial functions with the finite layer method to study the vibration of shear-deformable plates with intermediate line supports. Recently, Cheung and Kong [16] used the computed static beam functions under point loads to study the vibration of rectangular plates of varying complexity by the finite strip method.

In the present paper, a new set of admissible functions, which are the static solutions of a point supported beam under a series of sine loads, are developed for the vibration analysis of thin orthotropic rectangular plates continuous in one or two directions. The boundary conditions and the number and locations of point supports of the beam correspond to those of the plate in each direction. Each static beam function is composed of two parts: the polynomial function and a sine function. It should be mentioned that the order of the polynomials in this set of admissible functions is always lower than 4, so stable numerical computation can be achieved, especially for plates with a large number of intermediate line supports and/or when higher vibrating modes need to be known. Finally, some numerical results are given for rectangular plates with a number of intermediate line supports in one or two directions and compared with known values in the literature. It is shown that invariably a smaller number of terms of the admissible functions need to be used, and this implies lower computational cost.

2. A NEW SET OF ADMISSIBLE FUNCTIONS

Consider a uniform beam with J intermediate point supports under an arbitrary load $q(x)$ as shown in Figure 1. The length of the beam is l and the co-ordinates of the point supports of the beam are, respectively, x_j, $j = 1, 2, \ldots, J$.

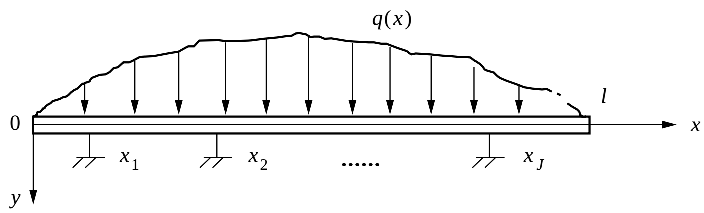

Figure 1. Point supported beam under arbitrary load $q(x)$.

The deflection $y(x)$ of the beam in the y direction should satisfy the differential equation

$$EI\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = \sum_{j=1}^{J} p_j \delta(x - x_j) + q(x), \qquad 0 < x < 1, \tag{1}$$

where EI is the flexural rigidity of the beam, p_j are the reactions of the jth point supports and $\delta(x - x_j)$ are the Dirac delta functions. Letting

$$\xi = \frac{x}{l}, \quad \xi_j = \frac{x_j}{l}, \quad P_j = \frac{p_j l^3}{EI}, \quad Q(\xi) = \frac{l^4}{EI} q(l\xi), \tag{2}$$

one has

$$\frac{\mathrm{d}^4 y}{\mathrm{d}\xi^4} = \sum_{j=1}^{l} P_j \delta(\xi - \xi_j) + Q(\xi), \qquad 0 < \xi < 1. \tag{3}$$

The arbitrary load $Q(\xi)$ can be expanded into a Fourier sine series as follows

$$Q(\xi) = \sum_{i=1}^{\infty} Q_i (i\pi)^4 \sin i\pi\xi, \qquad 0 < \xi < 1, \tag{4}$$

where Q_i are the unknown coefficients which may be decided uniquely if $Q(\xi)$ is given.

Considering the linear problems here, the solution $y(\xi)$ of equation (3) can take the form of

$$y(\xi) = \sum_{i=1}^{\infty} Q_i y_i(\xi). \tag{5}$$

Substituting equations (4) and (5) into equation (3), one obtains the total solution of equation (3) as

$$y_i(\xi) = \sum_{k=0}^{3} C_k^i \xi^k + \sum_{j=1}^{J} P_j^i \frac{(\xi - \xi_j)^3}{6} U(\xi - \xi_j) + \sin i\pi\xi, \tag{6}$$

where $C_k^i (k = 0, 2, 3)$ and $P_j^i (j = 1, 2, \ldots, J)$ are unknown coefficients, and $U(\xi - \xi_j)$ are the Heaviside functions. It should be noted that the order of the polynomials in the above equation is never higher than a cubic and is independent of the number of the intermediate point supports and the term number of the Fourier series.

For convenience, the support conditions along the ends of the beam are indicated by two capital letters. The letters C, S and F denote, respectively, clamped, simply supported and free ends. For the CC, CS, CF, SS beams and the SF beam with no less than one point support and the FF beam with no less than two point supports, the coefficients $C_k^i (k=0, 1, 2, 3)$ and $P_j^i (j=1, 2, \ldots, J)$ in equation (6) can be uniquely decided by the boundary conditions and the zero deflection conditions at the internal point supports of the beam and may be written in matrix form

$$\begin{bmatrix} A & D \\ T & G \end{bmatrix} \begin{bmatrix} C^i \\ P^i \end{bmatrix} = \begin{bmatrix} R^i \\ S^i \end{bmatrix}, \tag{7}$$

where A is a $J \times 4$ matrix, D is a $J \times J$ matrix and R^i is a $J \times 1$ matrix, and they refer to the values of the first series, the second series and the third term of equation (6) at the intermediate point supports of the beam, respectively. T is a 4×4 matrix, G is a $4 \times J$ matrix and S^i is a 4×1 matrix, and they refer to the values of the first series, the second series and the third term of equation (6) for the boundary conditions of the beam, respectively. C^i and P^i are the unknown coefficient matrices as follows:

$$C^i = [C_0^i C_1^i C_2^i C_3^i]^T, \qquad P^i = [P_1^i P_2^i, \ldots, P_J^i]^T. \tag{8}$$

Without losing generality and assuming that $\xi_j < \xi_k$ if $j < k$, the matrices D, A and R^i may be, respectively, written as

$$D = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 \\ \frac{(\xi_2-\xi_1)^3}{6} & 0 & 0 & \ldots & 0 \\ \frac{(\xi_3-\xi_1)^3}{6} & \frac{(\xi_3-\xi_2)^3}{6} & 0 & \ldots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ \frac{(\xi_J-\xi_1)^3}{6} & \frac{(\xi_J-\xi_2)^3}{6} & \ldots & \frac{(\xi_J-\xi_{J-1})^3}{6} & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 & \xi_i & \xi_1^2 & \xi_1^3 \\ 1 & \xi_2 & \xi_2^2 & \xi_2^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \xi_J & \xi_J^2 & \xi_J^3 \end{bmatrix}, \qquad R^i = \begin{bmatrix} -\sin i\pi\xi_1 \\ -\sin i\pi\xi_2 \\ \vdots \\ -\sin i\pi\xi_J \end{bmatrix}. \tag{9}$$

We use $t_{kl} (k=1, 2, 3, 4, l=1, 2, 3, 4)$, $g_{kj} (k=1, 2, 3, 4, j=1, 2, \ldots, J)$ and $s_k^i (k=1, 2, 3, 4)$ to represent the elements in matrices T, G and S^i, respectively. According to the boundary conditions of the beam, there are $t_{11}=1$, $t_{22}=1$, $s_2^i = -\mathrm{i}\pi$ for the beam with the C left end, $t_{11}=1$, $t_{23}=2$ for the beam with the S left end and $t_{13}=2$, $t_{24}=6$, $s_2^i=(i\pi)^2$ for the beam with the F left end. And

$t_{31}=t_{32}=t_{33}=t_{34}=1$, $t_{42}=1$, $t_{43}=2$, $t_{44}=3$, $g_{3j}=(1-\xi_j)^3/6$, $g_{4j}=(1-\xi_j)^2/2$, $s_4^i=-\mathrm{i}\pi(-1)^i$ for the beam with the C right end, $t_{31}=t_{31}=t_{33}=t={}_{34}=1$, $t_{43}=2$, $t_{44}=6$, $g_{3j}=(1-\xi_j)^3/6$, $g_{4j}=1-\xi_j$ for the beam with the S right end, $t_{33}=2$, $t_{34}=6$, $t_{44}=6$, $g_{3j}=1-\xi_j$, $g_{4j}=1$, $s_4^i=(i\pi)^3(-1)^i$ for the beam with the F right end. The other elements are all equal to zero. It is clearly shown that the first two rows of matrix G are always equal to zero. Solving the linear equation group (7) gives all the unknown coefficients.

If there is only an intermediate point support for the FF beam, the rigid rotation of the beam around the point support exists. In this case, the coefficient matrices C^i and P^i cannot be directly decided from equation (7). In this case, one may assume that

$$y(\xi)=\sum_{i=1}^{\infty}Q_i\bar{y}_i(\xi),$$

and then add the rigid rotation mode of the beam to the static beam functions by taking $\bar{y}_1(\xi)=\xi-\xi_1$ and $\bar{y}_i(\xi)=y_{i-1}(i\leqslant 2)$ which are the static beam functions of a SF beam with one corresponding intermediate point support. If there is no intermediate point support for the SF beam or the FF beam, the handling method is similar and has been described in the literature [17] in detail. It is important to note that matrices A and D in equation (7) are uniquely decided by the locations of the intermediate point supports of the beam, and matrices T and G in equation (7) are uniquely decided by the boundary conditions and the co-ordinates of the intermediate point supports of the beam. They are all independent of the series variable i, so only one inverse calculation to the coefficient matrix of equation (7) is needed for all i, so that the computational cost is greatly reduced.

3. THE EIGENFREQUENCY EQUATION

It is assumed that the plate under consideration lies in the x–y plane, is bounded by edges $x=0$, $x=a$ and $y=0$, $y=b$ and is of uniform thickness. The intermediate line supports are also assumed to lie orthogonal to the plate edges and to prevent motion in the z direction but to offer no resistance to normal rotation.

From the vibration theory of thin plates, the strain and kinetic energies of an elastic thin orthotropic plate in Cartesian co-ordinates are as follows:

$$U=\frac{1}{2}\int_0^a\int_0^b\left\{D_x\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2H\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+D_y\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right.$$
$$\left.-4D_{xy}\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}\mathrm{d}y\,\mathrm{d}x,\qquad T=\frac{1}{2}\int_0^a\int_0^b\rho h\left(\frac{\partial w}{\partial t}\right)^2\mathrm{d}y\,\mathrm{d}x,\quad (10)$$

where w is the deflection of the plate in the z direction, ρ is the material density, h is the plate thickness, D_x, D_y, H and D_{xy} are the flexural rigidities of the plate and $D_x=D_y=H=D$, $D_{xy}=(1-\nu)D/2$ for the isotropic case, where ν is the Poisson's ratio.

For free vibration of the plate, the deflection w may be expressed as

$$w(x, y, t) = W(x, y)\mathrm{e}^{i\omega t} \tag{11}$$

where ω is the radian eigenfrequency of vibration of the plate, t is the time and $i = \sqrt{-1}$. Assuming that

$$\xi = x/a, \qquad \eta = y/b \tag{12}$$

and that the variables in $W(x, y)$ are separable, the modal shape function $W(\xi, \eta)$ may be expressed in terms of a series as

$$W(\xi, \eta) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\varphi_m(\xi)\psi_n(\eta), \tag{13}$$

where $\varphi_m(\xi)$ and $\psi_n(\eta)$ are appropriate admissible functions which satisfy at least the geometrical boundary conditions and if possible, all the boundary conditions. A_{mn} are the unknown coefficients. Substituting equations (11), (12) and (13) into equation (10) and minimizing the total potential energy as follows

$$\frac{\partial}{\partial A_{mn}}(U_{\max} - T_{\max}) = 0 \tag{14}$$

leads to the eigenfrequency equation

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[C_{mnij} - \lambda^2 E_{ml}^{(0,0)} F_{nj}^{(0,0)}]A_{mn} = 0, \quad i, j = 1, 2, \ldots, \infty, \tag{15}$$

where

$$\begin{aligned}
C_{mnij} = {} & \frac{D_x}{H} E_{mi}^{(2,2)} F_{nj}^{(0,0)}/\gamma^4 + 4\frac{D_{xy}}{H} E_{mi}^{(1,1)} F_{nj}^{(1,1)}/\gamma^2 + \frac{D_y}{H} F_{mi}^{(0,0)} F_{nj}^{(2,2)} \\
& + \left(1 - 2\frac{D_{xy}}{H}\right)(E_{mi}^{(0,2)} F_{nj}^{(2,0)} + E_{mi}^{(2,0)} F_{nj}^{(0,2)})/\gamma^2, \quad m, n, i, j = 1, 2, 3, \ldots, \\
\gamma = {} & a/b, \qquad \lambda^2 = \rho h \omega^2 b^4/H, \\
E_{mi}^{(r,s)} = {} & \int_0^1 (\mathrm{d}^r\varphi_m/\mathrm{d}\xi^r)(\mathrm{d}^s\varphi_i/\mathrm{d}\xi^s)\,\mathrm{d}\xi, \\
F_{nj}^{(r,s)} = {} & \int_0^1 (\mathrm{d}^r\psi_n/\mathrm{d}\eta^r)(\mathrm{d}^s\psi_j/\mathrm{d}\eta^s)\,\mathrm{d}\eta.
\end{aligned} \tag{16}$$

The solution of equation (14) yields the eigenfrequencies of the vibration of the plate together with the coefficients for the mode shapes (13). The above analysis shows that the validity and accuracy of the solution depend entirely on the choice of the admissible functions $\varphi_m(\xi)$ and $\psi_n(\eta)$. Several approaches have been proposed for choosing $\varphi_m(\xi)$ and $\psi_n(\eta)$. However, it is not always easy to satisfy the simplicity, convergence and accuracy requirements. Here a set of static beam functions, which have been developed in the last section, are used as the admissible functions of the line supported rectangular plates, i.e.,

$$\varphi_m(\xi) = y_m(\xi), \qquad \psi_n(\eta) = y_n(\eta), \tag{17}$$

where $y_m(\xi)$ are the mth static beam functions which satisfy both the corresponding geometrical boundary conditions and the zero deflection conditions at the line supports of the plate in the x direction, and $y_n(\eta)$ are the nth static beam functions which satisfy those in the y direction. The integrated form of the stiffness matrix and the mass matrix of equations (14) may be given explicitly without any difficulty, if required.

4. SOME NUMERICAL RESULTS

In order to illustrate the accuracy, convergency and usefulness of the approach described above, some numerical results for free vibration of rectangular plates with several intermediate line-supports in one or two directions are reported and compared with the values available from other numerical methods. In all the computations $\nu = 0{\cdot}3$ is used. For brevity, four capital letters are used to represent the type of edges of the plate. The first two letters represent the boundary conditions of the plate in the x direction and the other represent those in the y direction. For uniformity of computation, the symmetry of structures is not considered.

In Figure 2 is shown the three-span continuous rectangular plate in the x direction. Both edges of the plate in the y direction are simply supported. The two intermediate line supports in the x direction are, respectively, at $x_1 = a/4$ and $x_2 = 3a/4$ and the side ratio of the plates is $\gamma = a/b = 4$. Two static beam functions in the y direction and seven static beam functions in the x direction are used. The first six eigenfrequency values obtained are listed in Table 1 for the plate with various boundary conditions. The first four mode shapes for the plate with clamped edges at $x = 0$ and $x = a$ are shown in Figure 3, which are very close to the exact mode shapes presented in the literature [7]. It can be seen from Table 1 that in all the numerical methods listed, the present results are closest to the exact values given by Azimi *et al.* [7], while the computational cost is the lowest as can be seen by the considerably smaller number of eigenfrequency equations used.

In Figure 4 is shown a two-direction, two-span continuous plate simply supported at all edges. The intermediate line supports in the x and y directions

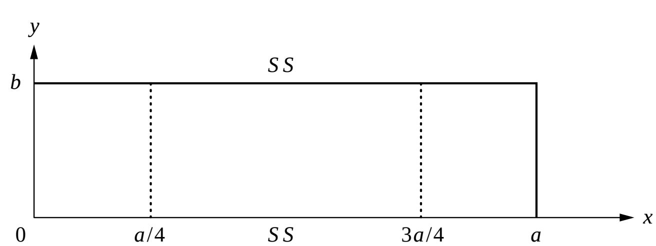

Figure 2. Three-span continuous rectangular plates in one direction.

TABLE 1

Eigenfrequency parameters $\lambda_i = \omega_i b^2 \sqrt{\rho h/D}$ $(i = 1, 2, \ldots, 6)$ *of a three-span continuous rectangular plate in one direction*

Edges	Source of results	Size of frequency equation	Mode sequence number					
			1	2	3	4	5	6
SS	Present	14	12·919	19·739	21·534	23·647	35·215	42·245
	Azimi *et al* [7]	exact	12·92	19·74	21·53	23·65	35·21	42·24
	Mizusawa and Kajita	256	12·921	19·741	21·551	23·682	35·415	
	Wu and Cheung [11]	52	12·92	19·74	21·55			
	Kim and Dickinson [12]†	36	12·930	19·739	21·594	23·812	35·401	42·268
	Liew and Lam [13]	80	12·924	19·739	21·531	23·653	35·283	42·254
CS	Present	14	12·938	20·097	22·643	26·506	35·605	42·247
	Azimi *et al*. [7]	exact	12·94	20·10	22·64	26·50	35·59	42·24
	Wu and Cheung [11]	52	12·94	20·10	22·67			
	Kim and Dickinson [12]†	36	12·972	20·118	22·916	26·915	36·628	42·307
	Liew and Lam [13]	80	12·961	20·114	22·866	26·514	36·164	42·286
CC	Present	14	12·957	20·816	25·648	27·128	35·980	42·249
	Azimi *et al*. [7]	exact	12·96	20·81	25·64	27·12	35·97	42·25
	Wu and Cheung [11]	52	12·96	20·83	25·69			
	Kim and Dickinson [12]†	36	12·967	20·828	25·684	27·262	36·170	42·296
	Liew and Lam [13]	80	12·963	20·814	25·654	27·179	35·998	42·263

† The symmetry is considered during computation.

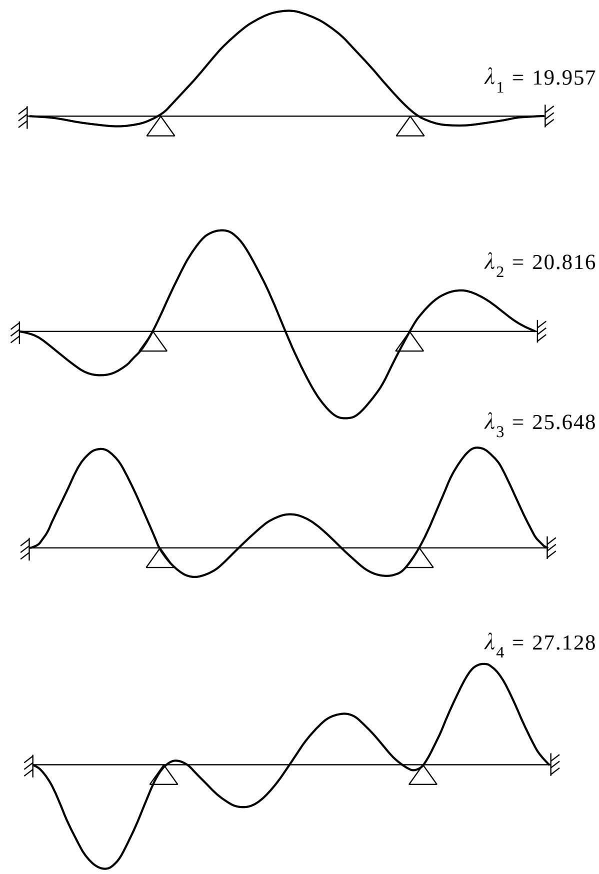

Figure 3. The first four mode shapes in the x direction for a rectangular plate with clamped edges at $x=0$ and $x=a$.

are at $x_1=\alpha a$ and $y_1=\beta b$, respectively. Two types of the plate are considered: (i) a square plate ($\gamma=1$) with various values of α and β and (ii) a rectangular plate ($\gamma=1{\cdot}5$) with $\alpha=\beta=1/\sqrt{3}$. Four static beam functions are used in each direction. The first six eigenfrequency values for the plates are listed in Table 2

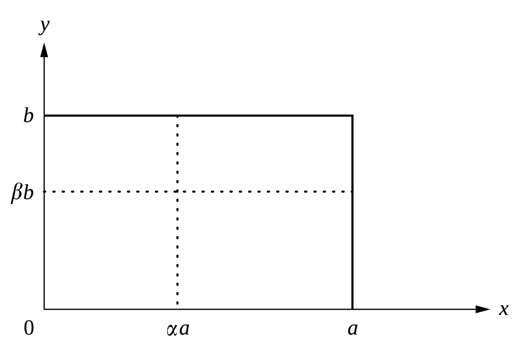

Figure 4. Two-span continuous rectangular plates in two directions.

TABLE 2

Eigenfrequency parameters $\lambda_i = \omega_i b^2 \sqrt{\rho h/D}$ $(i = 1, 2, \ldots, 6)$ *of a two-direction, two-span continuous rectangular plates simply supported at all edges*

Side ratio γ	Support location α	β	Source of results	Size of frequency equation	Mode sequence number 1	2	3	4	5	6
1	1/4	1/4	Present	16	42·605	96·762	96·878	151·18	176·91	176·91
			Kim and Dickinson [12]	36	42·844	97·437	97·531	152·58	178·59	178·59
			Liew and Lam [13]	80	42·731	96·682	96·899	151·60	176·00	176·00
			Zhou [14]	25	42·709	97·070	97·173	151·86	177·06	177·06
	1/4	1/2	Present	16	59·889	77·856	115·91	128·41	176·91	197·39
			Kim and Dickinson [12]	36		79·040	116·48	130·58	178·91	198·70
			Liew and Lam [13]	80	59·991	77·733	115·77	127·84	177·47	195·02§
			Zhou [14]	25	59·964	78·578	116·17	129·76	177·06	197·39
	1/2	1/2	Present	16	78·957	94·590	94·590	108·24	197·39	197·39
			Wu and Cheung [11]	72	78·96	94·68	94·72	108·44	197·40	198·96
			Kim and Dickinson [12]	36	78·958	95·911	95·911	110·81	199·02	199·02
			Liew and Lam [13]	80	78·958	94·826	94·826	108·41	197·50	197·50
			Zhou [14]	25	78·957	95·433	95·433	109·93	197·39	197·39
			Leissa [18]	exact	78·957	94·585	94·585	108·22	197·39	197·39
	0	0	Present†	16	27·057	60·551	60·797	92·886	114·62	114·76
			Kim and Dickinson [12]	36	27·887	62·484	62·723	95·995	114·81	118·54
			Liew and Lam [13]	80	27·055	60·543	60·791	92·854	114·49	114·64
			Leissa [18]‡	36	27·056	60·544	60·791	92·865	114·57	114·72
1·5	$1/\sqrt{3}$	$1/\sqrt{3}$	Present	16	49·031	62·910	83·895	91·303	96·307	123·42
			Kim and Dickinson [12]	36	49·293	63·925	85·322	94·445	98·712	128·15
			Takahashi *et al.* [10]		49·305	62·907	83·892	91·301	96·296	123·41

† $\alpha = \beta = 10^{-6}$ is taken in computation. ‡ Vibrating beam functions are used. § This value is obviously unreasonable; the exact value is 197·39, which can be obtained from the exact solution of simply supported square plate.

and compared with the results presented by Takahashi and Chishaki [10] using a sine series analytical solution, Wu and Cheung [11] using two-span vibrating beam functions in the finite strip method, Kim and Dickinson [12] using one-dimensional orthogonal polynomials, Liew and Lam [13] using two-dimensional orthogonal polynomials, Zhou [14] using modified vibrating beam functions in the Rayleigh–Ritz method and the results presented by Leissa [18]. Good agreement is observed for all cases, and invariably the present method always uses the smallest number of eigenfrequency equations. One may also find that when α and β approach zero, the edges $x=0$ and $y=0$ tend to become clamped supports, and such a rather severe limiting case can be reproduced nearly exactly by the present method. This shows the reliability and applicability of the present method and its superiority over other types of beam functions.

In Figure 5 is shown a two-direction, three-span continuous square plate ($a=b$). The two line supports in the x direction and the two line supports in the y direction are at $x_1=y_1=0{\cdot}35b$ and $x_2=y_2=0{\cdot}7b$, respectively. Three types of edge conditions are investigated: (i) all edges clamped (CC–CC); (ii) two adjacent edges clamped, the other two simply supported (CS–CS) and (iii) two opposite edges clamped, the other two simply supported (CC–SS). Four static beam functions are used in each direction. The first six eigenfrequency values of the plate are listed in Table 3 and compared with the results presented by Kim and Dickinson [12] using one-dimensional orthogonal polynomials and Zhou [14] using modified vibrating beam functions. Again good agreement is observed for all cases. The convergency study for the CC–CC case shows that the convergency rate of the present method is very rapid.

Next, a six-unequal-span continuous square plate ($a=b$) in two directions is investigated. The intermediate line supports in the x direction are at $x_1=0{\cdot}2a$, $x_2=0{\cdot}35a$, $x_3=0{\cdot}55a$, $x_4=0{\cdot}7a$ and $x_5=0{\cdot}8a$ and those in the y direction are at $y_1=0{\cdot}1b$, $y_2=0{\cdot}25b$, $y_3=0{\cdot}45b$, $y_4=0{\cdot}7b$ and $y_5=0{\cdot}9b$. It is clear that symmetry does not exist for this plate. Nine types of boundary conditions are considered. Seven static beam functions are used in each direction. The first eight eigenfrequency values are listed in Table 4.

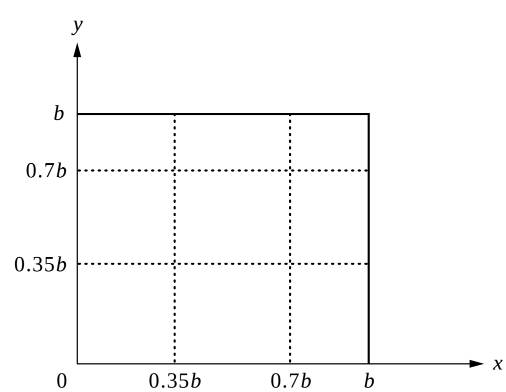

Figure 5. Three-span continuous square plate in two directions.

TABLE 3

Eigenfrequency parameters $\lambda_i = \omega_i b^2 \sqrt{\rho h / H}$ $(i = 1, 2, \ldots, 6)$ *of two-direction, three-span continuous square orthotropic plates with various boundary conditions*

Material property D_x/H; D_y/H	Edges	Source of results	Size of frequency equation	Mode sequence number 1	2	3	4	5	6
1; 1	CC–CC	Present	1	237·40					
			4	223·45	250·35	250·39	275·04		
			9	197·12	238·90	238·90	274·81	290·21	290·41
			16	197·12	237·64	237·64	272·75	287·72	287·92
		Kim and Dickinson [12]	36	198·55	243·27	243·32	282·31	297·20	297·20
		Zhou [14]	25	198·04	240·01	240·03	276·61	291·09	291·15
	CS–CS	Present	16	189·56	223·77	224·01	247·93	248·18	254·89
		Kim and Dickinson [12]	36	190·69	226·87	227·18	259·99	265·88	265·93
		Zhou [14]	25	189·95	226·25	226·53	251·86	251·98	259·42
	CC–SS	Present	16	183·33	208·69	226·04	244·11	247·79	277·76
		Kim and Dickinson [12]	36	184·34	212·77	231·39	256·24	262·44	286·99
		Zhou [14]	25	184·05	210·74	228·25	248·46	251·37	280·80
1·543; 4·81	CC–CC	Present	16	299·28	339·92	391·82	406·87	438·03	480·23
		Kim and Dickinson [12]	36	301·03	345·36	401·76	414·92	449·04	494·99
		Zhou [14]	25	300·30	342·08	394·16	409·21	441·42	484·22

TABLE 4

Eigenfrequency parameters $\lambda_i = \omega_i b^2 \sqrt{\rho h/D}$ ($i = 1, 2, \ldots, 8$) *of a two-direction, six-unequal-span continuous square plate with various boundary conditions*

	Mode sequence number							
Edges	1	2	3	4	5	6	7	8
SS–SS	479·93	495·62	531·41	608·35	620·50	643·44	651·79	654·97
CC–CC	519·58	596·93	612·60	645·07	680·21	711·31	723·97	743·14
SS–CC	480·18	495·87	531·64	610·66	622·77	647·89	653·93	659·34
FF–FF	250·95	253·53	288·12	291·26	306·12	309·01	433·73	435·14
SS–FF	472·79	488·67	501·04	510·21	515·93	524·79	525·29	551·58
CC–FF	513·21	540·47	549·36	591·71	607·44	616·42	624·55	631·31
CF–FF	253·13	290·42	308·24	434·90	506·87	513·44	540·67	549·55
CF–CF	253·42	298·77	429·34	506·23	513·50	541·59	592·32	617·61
SF–SF	253·45	298·85	428·24	472·95	502·32	506·05	524·86	552·12

The last treated problem is a one-direction, ten-unequal-span continuous rectangular plate with aspect ratio $\gamma = a/b = 10$. The intermediate line supports are all in the x direction at $x_1 = 0{\cdot}1a$, $x_2 = 0{\cdot}2a$, $x_3 = 0{\cdot}35$, $x_4 = 0{\cdot}45a$, $x_5 = 0{\cdot}5a$, $x_6 = 0{\cdot}6a$, $x_7 = 0{\cdot}7a$, $x_8 = 0{\cdot}8a$ and $x_9 = 0{\cdot}9a$. Eight types of boundary conditions are considered. In the computation, 15 static beam functions in the x direction and five static beam functions in the y direction are used. The first ten eigenfrequency values are listed in Table 5. It is shown that the present method is also suitable for plates with a large number of intermediate line supports.

It is seen that the number of the static beam functions used in the analysis is concerned mainly with the number of the line supports and is also concerned with the locations of the line supports and the aspect ratio of the rectangular plate, which can be determined by the convergent study for a practical problem. Many examples show that only a small number of terms of the static beam functions will give enough accuracy.

5. CONCLUSIONS

A new set of admissible functions which are the static solution of a point supported beam under a series of sine loads has been described and applied to study the free vibration of one-direction or two-direction continuous rectangular plates with various classical bound conditions in the Rayleigh–Ritz method. The vibration characteristics of the plates can be solved in an easy and unified manner. The basic concept to develop the set of static beam functions is theoretically sound but relatively simple, and requires no complicated mathematical functions. Several numerical examples have been analyzed. The present results are compared with values obtained by other numerical methods in the literature. It can be seen that rapid convergency and good accuracy are achieved with a small number of terms of the static beam functions. Moreover, because the order of the polynomials in this set of static beam functions is

TABLE 5

Eigenfrequency parameters $\lambda_i = \omega_i b^2 \sqrt{\rho h/D}$ $(i = 1, 2, \ldots, 10)$ *of a one-direction, ten-unequal-span continuous rectangular plate with various boundary conditions*

Edges	Mode sequence number									
	1	2	3	4	5	6	7	8	9	10
SS–SS	15·441	19·958	20·422	21·457	21·809	24·416	24·784	26·137	27·582	32·355
CC–CC	25·782	29·562	30·230	30·900	31·333	33·424	33·871	34·963	35·736	39·167
FF–CF	6·1674	6·1794	9·1453	14·023	15·027	16·831	17·593	20·476	21·923	25·006
FF–SS	12·230	12·236	15·454	20·749	21·562	23·150	23·567	25·976	27·138	32·327
FF–CS	17·238	17·244	19·856	24·519	25·226	26·638	26·987	29·141	30·185	35·146
CC–CF	9·1189	13·862	14·936	16·170	16·893	20·004	20·841	22·405	23·677	26·638
FF–CC	23·741	23·746	25·787	29·680	30·274	31·484	31·772	33·635	34·545	39·090
SS–CF	9·1003	13·044	13·639	14·946	15·431	18·487	18·985	20·745	22·470	26·483

always lower than 4 and is independent of both the number of intermediate supports and the term number of the Fourier series to be used, the numerical instability of high order polynomials in the numerical computation is therefore avoided. Therefore, the present method is especially suitable for the plate problem with a large number of the intermediate line supports and/or when higher vibrating modes need to be calculated.

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